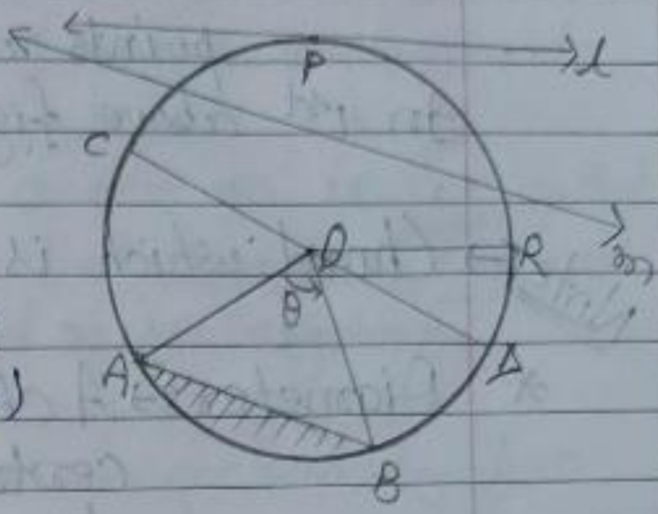
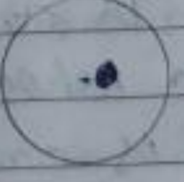
# Circle

O ⇒ centre
OR ⇒ Radius
AB ⇒ chord
$\widehat{AB}$ ⇒ Arc
ABO ⇒ Sector
Area inclosed AB ⇒ Segment
$\angle AOB$ ⇒ Central angle ($\theta$)
CD ⇒ Diameter
Line $l$ ⇒ Tangent
~~Line~~ Point P ⇒ point of contact
Line m ⇒ secant.




* Circle :→ A set of all points on a plane which are equidistant from a fixed given point.

→ Here, fixed given point (O) is called centre of a circle.



* Radius :→ Line segment joining from centre to any point on a circle. is called radius.

In 1st above figure OR is radius.

→ It is denoted by small 'r'.

Note :→ A circle whose centre 'O' and radius 'r' is denoted by 'C(O, r)'.

*. Chord:→ Line segment joining two distinct points on a circle is called chord.
In 1st above figure $\overline{AB}$ is chord.

Note:→ Chord which is nearer to centre is larger.

* Diameter:→ A chord which passes through centre of a circle is called diameter.
→ Diameter is the longest chord of a circle.
→ Diameter is twice of radius.

* Arc:→ A continuous part of a circle is called arc.
→ An arc whose length is less than the semi circle is called minor arc.
→ An arc whose length is more than the semi circle is called major arc.
→ It is denoted by '$\widehat{AB}$'
* Segment:→ Part of circle enclosed by a chord and corresponding arc is called segment.

*→ Minor and major segment:→ A segment enclosed by a chord and minor arc is called minor segment

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A segment enclosed by a chord and a major arc is called major ~~as~~ segment.

Or

A segment enclosed centre in it is called major segment and A segment ~~does~~ dose not enclosed centre in it is called minor segment.

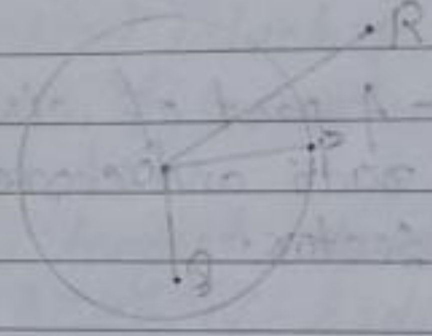
* Sector :- A part of circle enclosed by two radii and corresponding arc is called sector.

*→ Minor and major sector :→ A sector whose central angle is less than 180° is called minor sector.
And A sector whose central angle is more than ~~one~~ 180° is called major sector.

Note :→ Circles divides the plane in three parts.
(i) Interior of a circle.
(ii) Exterior of a circle.
(iii) Boundary of a circle.

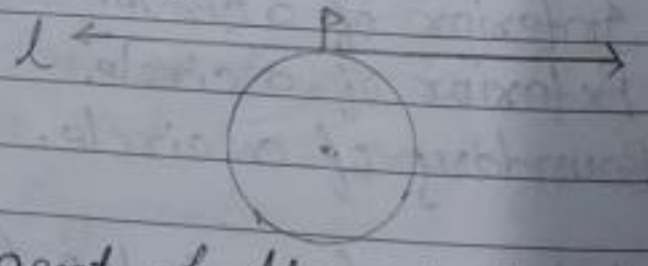
*→ (i) Interior of a circle :- The region which consists all point lie inside of the circle.

* (ii) Exterior of circle :- The region which consists all the points lies outside of the circle.
* (iii) Boundary of a circle :- The region which consists all the points lies on the circle.



→ (i) $OQ < r$, then Q must lie interior of a circle.
→ (ii) $OP = r$, then P must lie on the circle.
→ (iii) $OR > r$, then R must lie exterior of a circle.

* Tangent of a circle :- A line which intersect the circle at a point is called tangent of a circle.

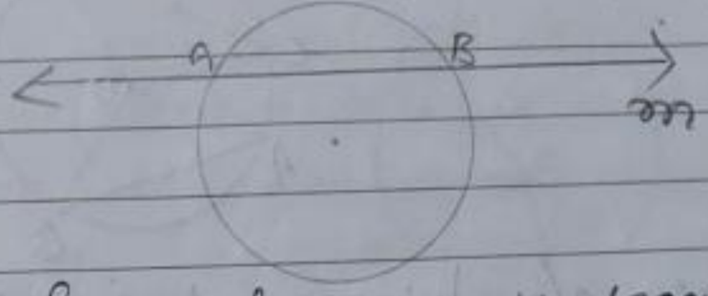


→ line $l$ is tangent of the circle.

Note:- Common point of circle and tangent is called "point of contact".
→ Here, P is point of contact.

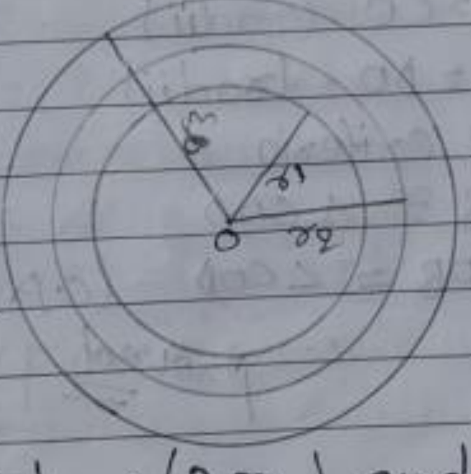
* Secant of a circle :- A line which intersect the circle at two distinct point is called secant of a circle.

→ line m intersect at two distinct point A and B.



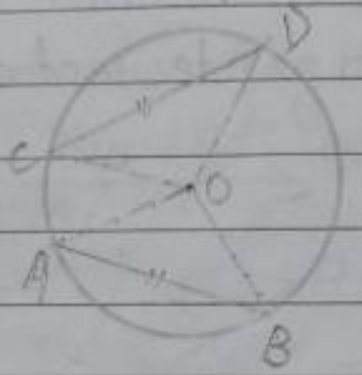

In given figure, line m is secant of the circle.

* Concentric circle :- Two or more circles having a common centre are called concentric circle.




Here, $c(O, r_1)$, $c(O, r_2)$ and $c(O, r_3)$ are concentric circles.

Theorem 1st :→ Equal chords of a circle subtend equal angle at centre.




Given :→ $c(o, r)$ is a circle in which chord $AB = CD$

To prove :→ $\angle AOB = \angle COD$

Proof :→ In $\Delta AOB$ and $\Delta COD$

$AB = CD$ (given)

$AO = CO$ (radii)

$BO = DO$ (radii)

By S.S.S. criteria

$\Delta AOB \cong \Delta COD$

$\therefore \angle AOB = \angle COD$ (C.P.C.T)

proved

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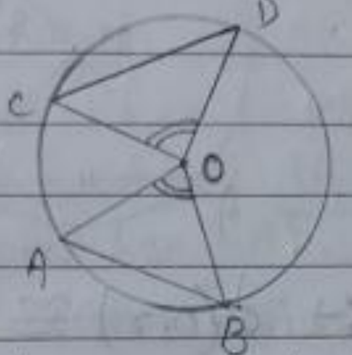
**Theorem 2nd** :→ (converse of previous theorem 1st)
:→ If two chord of a circle subtend equal angles at centre, then the chords are equal.




Given :→ $c(o, r)$ is a circle in which AB and CD are two chords such that $\angle AOB = \angle COD$

To prove :→ $AB = CD$

proof :→ for $\Delta AOB$ and $\Delta COD$

$AO = CO$ (radii)

$\angle AOB = \angle COD$ (given)

$BO = DO$ (radii)

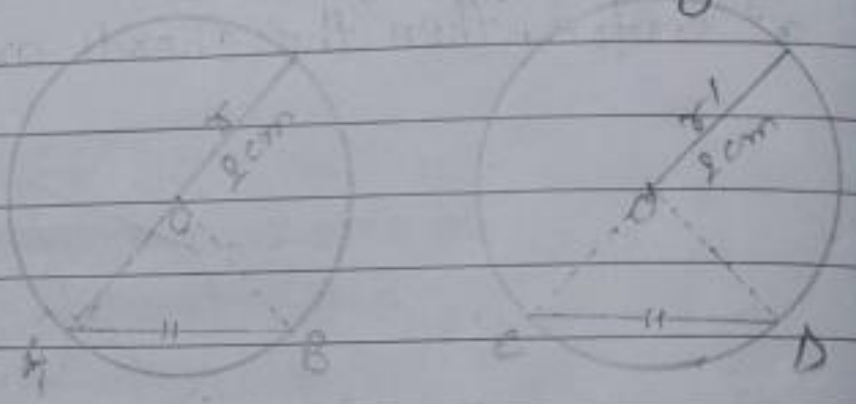
By S.A.S. criteria.

$\Delta AOB \cong \Delta COD$

$\therefore AB = CD$ (C.P.C.T)

proved

Note:→ Theorem:→ Equal chords of two congruent circles subtend equal angles at centre.




Given:→ $c(O, r) \cong c(O', r')$ in which chord $AB = CD$.

To prove:→ $\angle AOB = \angle COD$.

proof:→ $AO = OC$ (radius of two congruent circle)

$OB = OD$ ( ,, )

$AB = CD$ (given)

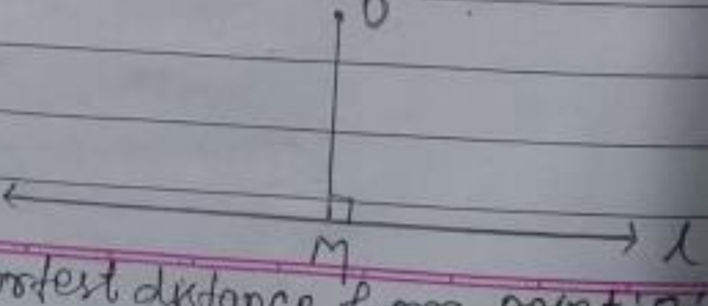
By S.S.S. criteria

$\Delta AOB \cong \Delta COD$.

$\therefore \angle AOB = \angle COD$ (C.P.C.T)

proved

Note:→ From a given point to a given line, perpendicular distance is shortest distance between point and line.




→ OM is shortest distance from point 'O' to line 'l'

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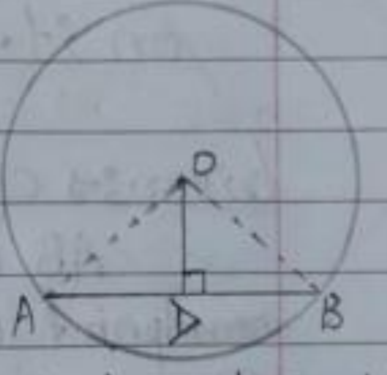
**Theorem 3rd** :→ Perpendicular from centre of a circle to the chord, bisect the chord.




Given :→ $c(O, r)$ is a circle in which $OD \perp AB$.

To prove :→ $AD = BD$

Construction :- Join AO and BO

proof :→ In right angle triangle AOD and BOD

$AO = BO$ (radii) (hypotenuse)

$OD = OD$ (common)

By. R.H.S criteria.

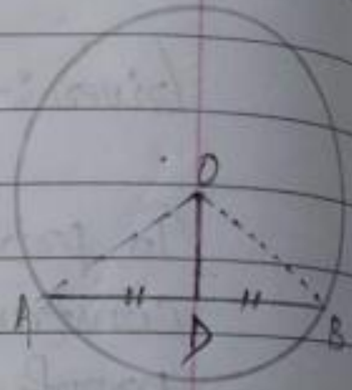
$\Delta AOD \cong \Delta BOD$

$\therefore AD = BD$ (C.P.C.T)

proved

**Theorem 4th** :→ (converse of previous theorem 3rd)

:→ Line segment joining from centre of a circle to midpoint of a chord is perpendicular on it.




Given :→ $C(O, r)$ is a circle in which AB is chord and D is midpoint of chord AB.

To prove :→ $OD \perp AB$.

Construction :→ Join AO and BO.

Proof :→ In $\triangle AOD$ and $\triangle BOD$

$OA = OB$ (radii)

$OD = OD$ (common)

$AD = BD$ (given)

By S.S.S criteria.

$\triangle AOD \cong \triangle BOD$

$\therefore \angle ADO = \angle BDO$ (C.P.C.T) eqn — (1)

By linear pair

$\angle ADO + \angle BDO = 180°$

$\angle ADO + \angle ADO = 180°$ { From eqn — (1) }

$2\angle ADO = 180°$

$\angle ADO = 90°$

$\therefore OD \perp AB$

proved

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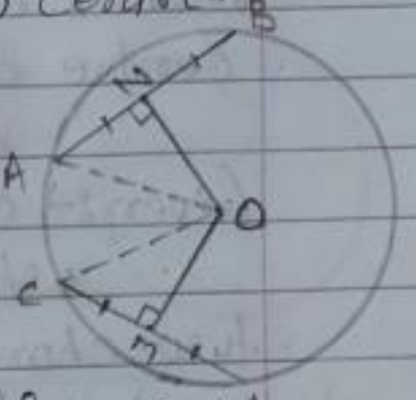
Theorem $5^{th}$ :- Equal chord of a circle are equidistant from centre.




Given :- $C(O, r)$ is a circle in which AB and CD are two equal chords $(AB = CD)$

To prove :- Chords are equidistant from centre.

or, $OM = ON$.

Construction :- Join AO and CO.

proof :- $AB = CD$ (given)

$\frac{1}{2}AB = \frac{1}{2}CD$ (multiplying $\frac{1}{2}$ both side)

$AN = CM$ {perpendicular from centre to chord, bisect the chord}

In $\triangle COM$ and $\triangle AON$

$AN = CM$

$AO = CO$ (radii) (hypotenuse)

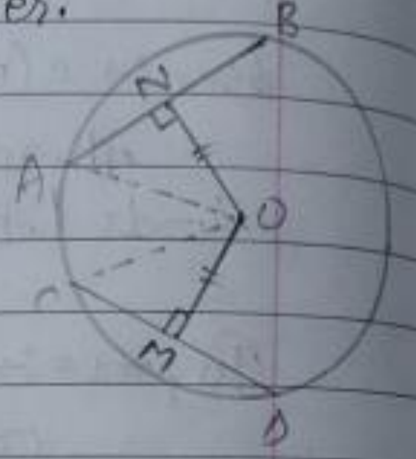
By R.H.S criteria.

$\triangle COM \cong \triangle AON$

$\therefore OM = ON$ (C.P.C.T)

proved

Theorem 6th :→ (converse of previous theorem 5th :→ Two chords which are equidistant from centre are equal to each other.




Given :→ $c(o, r)$ is a circle in which AB and CD are two chords equal distance from centre. (ON = OM)

To prove :→ AB = CD

Construction :→ Join AO and CO

proof :→ In $\triangle COM$ and $\triangle AON$

ON = OM (given)

AO = CO (radii) (hypotenuse)

$\angle CMO = \angle ANO$ (90°)

By R.H.S criteria.

$\triangle COM \cong \triangle AON$.

$\therefore$ CM = AN.

2CM = 2AN (multiplying 2 both side)

~~AB = CD~~

CD = AB proved

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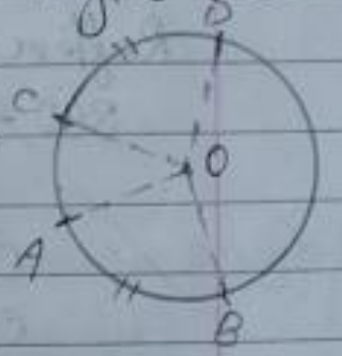
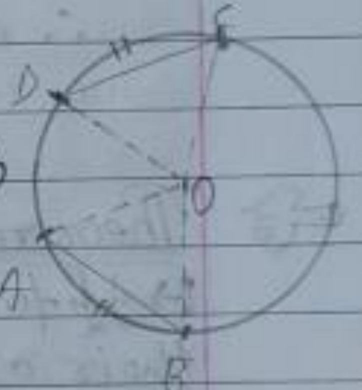
**Note:** If two arcs of a circle and are congruent then their corresponding central angles are equal.

If $\widehat{AB} \cong \widehat{CD}$, then $\angle AOB = \angle COD$.




**Theorem 7th:** If the two arcs of the circle are congruents then their corresponding chords are equal.




Given: $C(O, r)$ is a circle in which $\widehat{AB}$ and $\widehat{CD}$ are two arc such that $\widehat{AB} \cong \widehat{CD}$

To prove: $AB = CD$

Construction: $OA$, $OB$, $OC$ and $OD$

Proof: Case $1^{st}$: If $\widehat{AB}$ and $\widehat{CD}$ are minor arcs.

$\widehat{AB} \cong \widehat{CD}$ (given)

$\therefore \angle AOB = \angle COD$

$OB = OD$ (radius)

$OA = OC$ (radius)

By S.A.S criteria.

$\triangle AOB \cong \triangle COD$

$\therefore AB = CD$ (C.P.C.T) proved

Case II$^{nd}$ :→ when $\overparen{AB}$ and $\overparen{CD}$ are major arc, $\overparen{AB} \cong \overparen{CD}$

Reflex $\angle AOB$ = Reflex $\angle COD$

$\cancel{360} + \angle AOB = \cancel{360°} + \angle COD$

$\angle AOB = \angle COD$.

$OA = OC$

$OB = OD$

By S.A.S criteria.

$\therefore \Delta AOB \cong \Delta COD$.

$\therefore AB = CD$ (C.P.C.T)

proved

Theorem 8$^{th}$ :→ (Converse of previous theorem 7$^{th}$)

:→ If two chords of a circle are equal then their corresponding arcs (minor, major or semicircular) are congruent


Given :→ $C(O, r)$ is a circle in which AB and CD are two chords. Such that $AB = CD$.

To prove :→ $\overparen{AB} \cong \overparen{CD}$

Construction :→ Join OA, OB, OC and OD.

Proof :→ Case I$^{st}$ :→ For minor arcs.

In $\Delta AOB$ and $\Delta COD$

$AB = CD$ given.

$OA = OC$ } (radius)

$OB = OD$

By S.S.S. criteria.

$\triangle AOB \cong \triangle COD$.

$\therefore \angle AOB = \angle COD$

$\therefore \overset{\frown}{AB} \cong \overset{\frown}{CD}$

proved

Case II^nd :→ when $\overset{\frown}{AB}$ and $\overset{\frown}{CD}$ are major arcs.

In $\triangle AOB$ and $\triangle COD$.

$AB = CD$ (given)

$\left.\begin{array}{l} OA = OC \\ OB = OD \end{array}\right\}$ radius

By S.S.S. criteria.

$\triangle AOB \cong \triangle COD$.

$\therefore \angle AOB = \angle COD$.

$360 - Ref.\angle AOB = 360 - Ref.\angle COD$

$Ref\angle AOB = Ref\angle COD$

$\therefore$ Major $\overset{\frown}{AB} \cong$ Major $\overset{\frown}{CD}$

proved

Case III^rd :→ for Arcs of semi circles.

If $\overset{\frown}{AB}$ and $\overset{\frown}{CD}$ are arcs of semi-circle then AB and CD are Diameter.


Given :→ $AB = CD$

$\frac{1}{2}AB = \frac{1}{2}CD$.

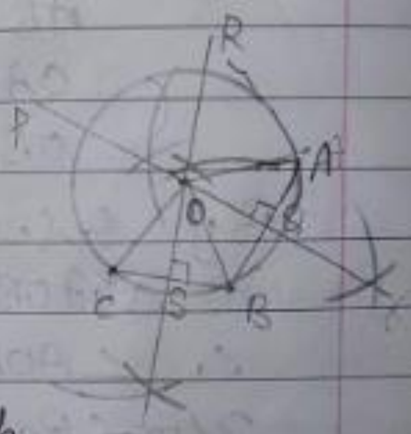
$OA = OC$.

radius are equal then semi circles with radius OA and OC are congruent.

$\widehat{AB} \cong \widehat{CB}$

proved

√.√.I Theorem 9th :→ There is one and only one circle passes through three non collinear point.




Given :→ A, B, and C are three non-collinear points.

To prove :→ One and only one circle passes through A, B, and C.

Construction :→ Join AB and BC

proof :→ Draw PQ and RS, Perpendicular bisector of AB and BC, respectively which intersect at O. Join OA, OB and OC.

In $\Delta OCS$ and $\Delta OBS$.

$CS = BS$ (bisector)

$\angle OSC = \angle OSB$ $(90^\circ)$

$OS = OS$ (common)

By S.A.S criteria.

$\Delta OCS \cong \Delta OBS$.

$\therefore OC = OB$ (C.P.C.T) —— (1)

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Similarly

$OB = OA$ —— (ii)

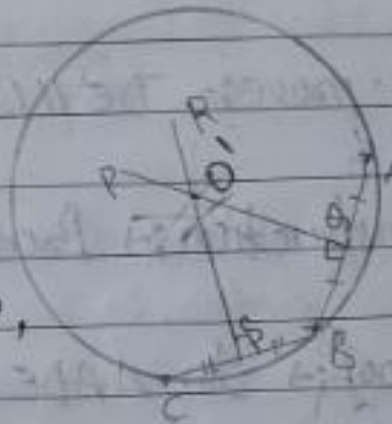
From (i) and (ii)

$OA = OB = OC$

with O as centre and $OA = OB = OC$ as radius. Draw a circle $c(O, r)$ which passes through A, B and C

Proved

**For uniqueness**




If possible, let another circle $c(O', r')$ also passes through point A, B, and C.

$\therefore$ AB and BC are chords of a circle $c(O', r')$.

PQ and RS are perpendicular bisector of AB and BC.

then point of intersection of PQ and RS is centre $O'$.

But given PQ and RS intersect at O.

we know that two lines intersect at one and only one point.

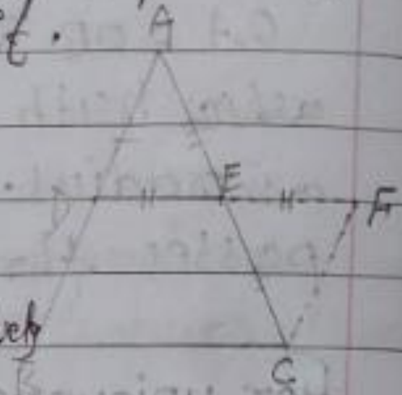
$\therefore$ $O'$ coincide on O.

$\therefore$ $O'A = OA$ $\}$ $\therefore$ $c(O', r')$ coincide on $c(O, r)$

$r' = r$ $\}$ $\therefore$ Both circle are same. proved

V.V.I ** Midpoint Theorem of Triangle :- Line segment joining midpoint of two sides of a triangles is parallel to third side and half of it.

B.M




Given :- ABC is a triangle in which D and E is midpoint of AB and AC respectively

To prove :- $DE \parallel BC$ and $BC = 2\,DE$

Construction :- Produce DE to F such that $DE = EF$. Join FC.

Proof :- In $\Delta ADE$ and $\Delta EFC$.

$DE = EF$ (construction)

$AE = EC$ (given)

$\angle AED = \angle FEC$ (V.O.A)

By S.A.S criteria.

$\Delta ADE \cong \Delta EFC$

$\therefore \angle F = \angle ADE$ and $FC = AD$ {C.P.C.T}

$\angle F$ and $\angle ADE$ are pair of Alt. interior angle.

$\therefore DB \parallel FC$ — (i)

$FC = AD$ (from C.P.C.T)

$AD = DB$ (given)

$\therefore DB = FC$ — (ii)

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From eqn (i) and (ii)

If one pair of opposite sides are parallel and equal then quadrilateral is ||gm.

$\therefore$ DFCB is ||gm.

DF || BC

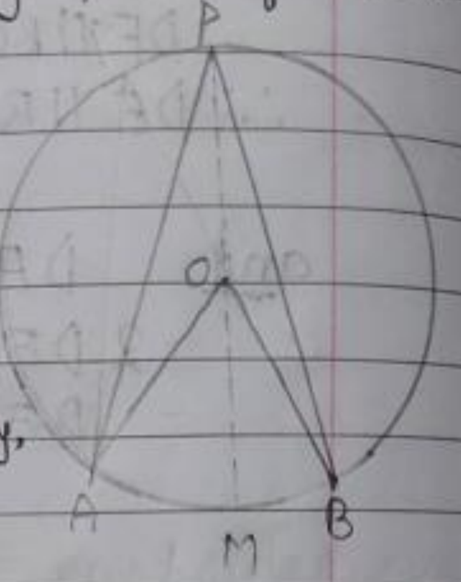
$\therefore$ DE || BC proved

and DF = BC

2 DE = BC.

$DE = \frac{1}{2} BC$. proved

Theorem $10^{th}$ :→ The angles subtanded by an arc of a circle at the centre is double then the angles subtanded by it at any point on the remaining part of circle.




Given :→ $c(o, r)$ is a circle in which $\overset{\frown}{AB}$ is an arc $\angle AOB$ and $\angle APB$ are angles on the centre and on the circle poit p respectively.

To prove :→ $\angle AOB = 2\angle APB$

Construction :→ Draw a line PM which passes through O.

proof :→ In $\Delta APO$

$OP = OA$

$\angle OPA = \angle OAP = x$ (let)

using exterior angle property of triangle,

$\angle AOM = \angle OPA + \angle OAP = x + x = 2x$

In $\Delta OPB$

$OP = OB$

$\angle OPB = \angle OBP = y$ (let)

using exterior angle property of triangle

$\angle BOM = \angle OPB + \angle OBP = y + y = 2y$

$\angle APB = x + y$ —— (1)

$\angle AOB = \angle AOM + \angle BOM$

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$\angle AOB = 2x + 2y$

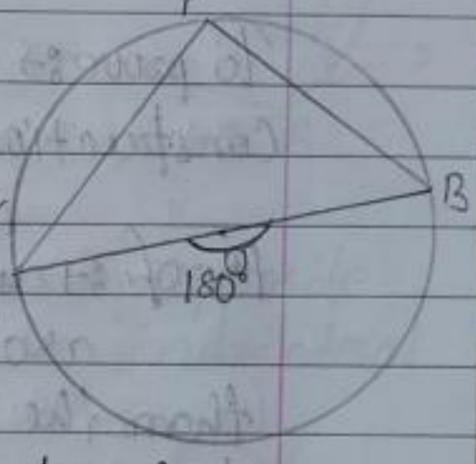
$\angle AOB = 2(x+y)$

$\angle AOB = 2(\angle APO + \angle BPO)$

$\angle AOB = 2\angle APB$

proved

**Theorem 11th :→** Angle on semicircle is right angle.




Given :- $c(o,r)$ is a circle in which AB is a diameter $\angle APB$ is an angle on semicircle

To prove :- $\angle APB = 90°$

proof :→ we know that diameter AB is a straight line.

$\therefore \angle AOB = 180°$

$\angle APB = \frac{1}{2}\angle AOB$ {Angle subtand at centre by an arc is double than the angle subtand by it on circle}

$\angle APB = \frac{1}{2} \times \cancel{180}^{90°}$

$\angle APB = 90°$

proved

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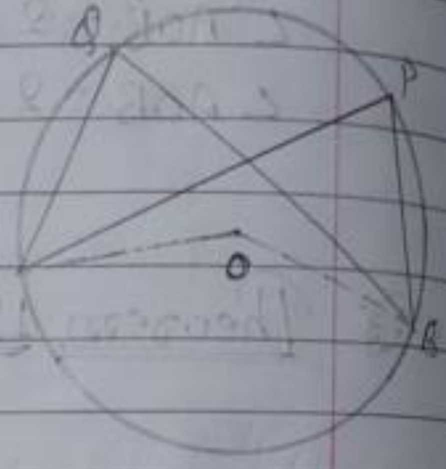
Theorem 12th :→ Angles in the same segment of a circle are equal.




Given :→ c(o,r) is a circle $\angle APB$ and $\angle AQB$ are angles on same segment of the circle.

To prove :→ $\angle APB = \angle AQB$.

Construction :→ Join AO and BO

Proof :→ we know that angle subtended by an arc of a circle at centre is double than the angle subtended by it at any point on the circle.

$$\angle APB = \frac{1}{2} \times \angle AOB \quad \text{— (i)}$$

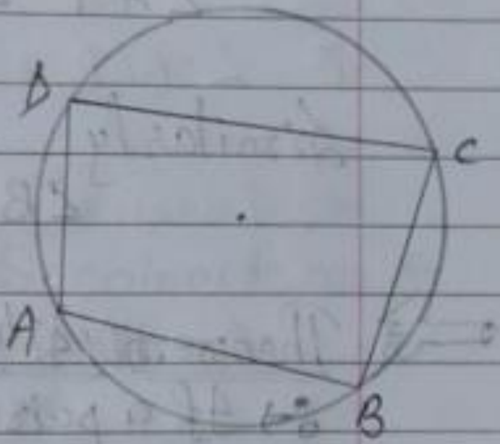
$$\angle AQB = \frac{1}{2} \times \angle AOB \quad \text{— (ii)}$$

R.H.S of both eqn is equal the L.H.S is must be equal.

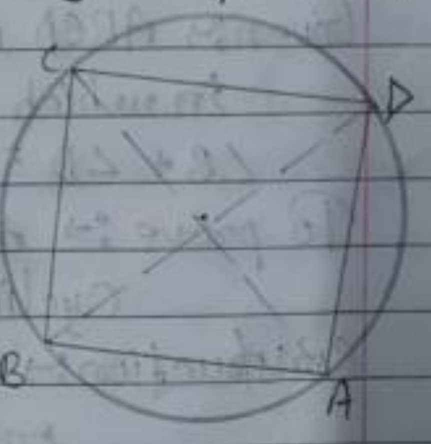
$\therefore \angle APB = \angle AQB$ [from eqn (i) and (ii)]

proved

** Cyclic quadrilateral :→ A quadrilateral whose all four vertices lies on the circle is called cyclic quadrilateral.



Theorem 13th :→ The sum of pair of opposite angles of a cyclic quadrilateral is 180°.



Given :→ ABCD is a cyclic quadrilateral.

To prove :→ (i) $\angle A + \angle C = 180°$

(ii) $\angle B + \angle D = 180°$

Construction :→ Join AC and BD

Proof :→ we know that angles on same segment are equal.

$\angle BAC = \angle BDC$ —— (i)

$\angle CAD = \angle CBD$ —— (ii)

In $\Delta BCD$

$\angle BDC + \angle CBD + \angle BCD = 180°$

Putt in the value of eqn - (i) and (ii)

$\angle BAC + \angle CAD + \angle BCD = 180^\circ$

$\angle A + \angle C = 180^\circ$

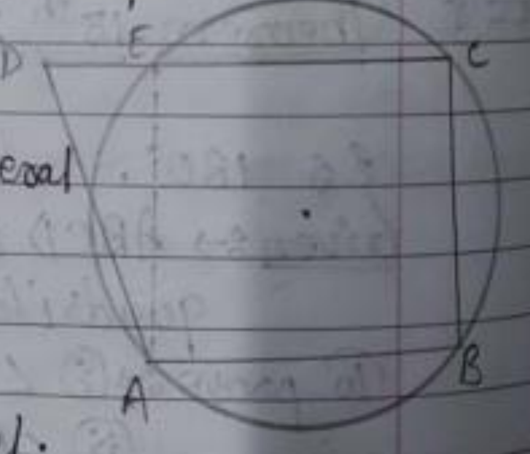
proved

Similarly

$\angle B + \angle D = 180^\circ$ proved

**Theorem 14th** :→ (Converse of previous theorem 13th)

:→ If a pair of opposite angles of quadrilateral is Supplimentry then the quadrilateral is cyclic.



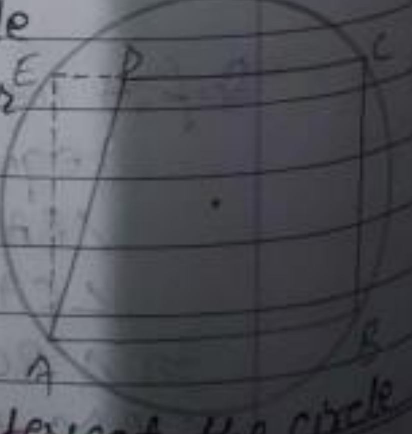

Given :→ ABCD is a quadrilateral in which

$\angle B + \angle D = 180^\circ$ —— (1)

To prove :→ ABCD is a cyclic quadrilateral.

Construction :→ we know that there is one and only one circle passes through three non-collinear point.

or




Draw a circle $c(o,r)$ passes through A, B and C.

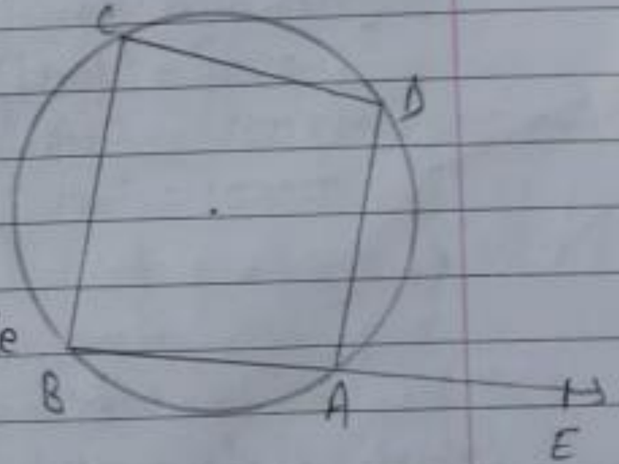
If possible, let D not lies on $c(o,r)$ and CD or produced CD intersect the circle

Proof:→ ABCE is a cyclic quadrilateral. ~~opposite~~
Opposite angles of cyclic quadrilateral is supplementary
$\therefore \angle B + \angle E = 180°$ —— (ii)
from (i) and (ii)

$$\angle B + \angle D = \angle B + \angle E$$
$$\angle D = \angle D$$

it is only possible when D coincide on E.
$\therefore$ ABCD is a cyclic quadrilateral.
proved

Theorem 15th :→ Exterior angle property of cyclic quadrilateral:→ If any one side of a cyclic quadrilateral be produce then exterior angle so formed is equal to interior opposite angle.




Given:→ ABCD is a cyclic quadrilateral and side BA produced to E such that exterior angle $\angle DAE$ formed.

To prove:- $\angle BCD = \angle DAE$.

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Proof :→ opposite angle of cyclic quadrilateral is supplementary.

$\therefore \angle BAD + \angle BCD = 180° \quad \text{—} (1)$

B $\angle BAD + \angle DAE = 180° \quad \text{—} (ii)$ {linear pair}

from (i) and (ii)

$\cancel{\angle BAD} + \angle BCD = \cancel{\angle BAD} + \angle DAE$

$\angle BCD = \angle DAE$

proved